

Illustrating Student Achievement Using National Assessment of Educational Progress Questions: Grade 8 Expressions and Equations Domain

The Montana Office of Public Instruction (OPI) adopted new standards for language arts and mathematics in November 2011. The new standards will be implemented in the 2013-2014 school year with the Smarter Balanced (SBAC) assessment taking place in the spring of 2014.

This document uses National Assessment of Education Progress (NAEP) questions that seem to have a close alignment with the new standards to illustrate or suggest current levels of student achievement for the new standards. It is not intended to make any predictions about how students will do on a new assessment but may have instructional implications in terms of showing students' strengths and weaknesses. NAEP releases some items after each NAEP administration; performance data is given for the nation and states for each released item. Since 2003, every state has participated in the grade 4 and grade 8 NAEP mathematics and language arts assessments, which are given every other year. SBAC released practice tests matching the Expressions and Equations domain have been included in this document as another example to illustrate the standards. There are no NAEP 2013 released questions as examples but these questions may be accessed via the [NAEP Questions Tool \(NQT\)](#).

This work has been made available through the **National NAEP Year Projects** (NNYP). This document parallels the work of Alaska's NAEP state coordinator. The following jurisdictions have made this information possible: Alaska, Iowa, New York, Florida, Oregon and the District of Columbia. For more information and resources, please visit:

- [Alaska Department of Education](#)
- [Iowa Department of Education](#)
- [NYC Department of Education](#)
- [Florida Department of Education](#)
- [Oregon Department of Education](#)
- [District of Columbia](#)
- [AIR: Examining the Content and Context of the Common Core State Standards: A First Look at Implications for the National Assessment of Educational Progress](#)



Montana
Office of Public Instruction
Denise Juneau, State Superintendent

A note about NAEP performance: NAEP rates multiple-choice or constructed-response questions scored either right or wrong as “easy” if answered correctly by 60% or more of students, “medium” is answered correctly by 40 to 59%, or “hard” if answered correctly by fewer than 40%.

Montana Common Core Standards (MCCS):

Work with radicals and integer exponents.

- 8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both standard notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

- 8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Analyze and solve linear equations and pairs of simultaneous linear equations.

- 8.EE.7. Solve linear equations in one variable.
 - a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - b. Solve linear equations with rational coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.

Analyze and solve linear equations and pairs of simultaneous linear equations.

- 8.EE.8. Analyze and solve systems of linear equations.
 - a. Show that the solution to a system of two linear equations in two variables is the intersection of the graphs of those equations because points of intersection satisfy both equations simultaneously.
 - b. Solve systems of two linear equations in two variables and estimate solutions by graphing the equations. Simple cases may be done by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
 - c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Define, evaluate, and compare functions.

- 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 6.EE.3. Apply the properties of operations as strategies to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Reason about and solve one-variable equations and inequalities.

- 6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

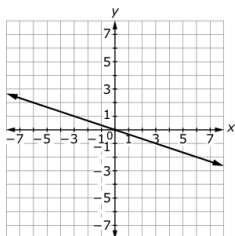
For more information on the MCCS- Grade Level Standards by Domain and Cluster, please visit: http://opi.mt.gov/Curriculum/montCAS/MCCS/index.php?gpm=1_4

Year	Grade	Block	#	Type	Difficulty	Content Area	% Correct	Item	Description	Iowa CCSS Code	Alaska CCSS Code
2003	8	7	16	MC	Medium	Number sense, properties, and operations	41.42	Item1	Identify decimal for scientific notation (calculator available)	8.EE.4	8.EE.4
2003	8	7	19	ECR	Hard	Algebra and functions	22.98	Item2	Solve problem using informal algebra (calculator available)	8.EE.8	8.EE.8
2003	8	10	13	SCR	Hard	Algebra and functions	34.3	Item3	Solve problem involving two linear relationships	8.EE.8	8.EE.8
2005	8	12	3	MC	Medium	Algebra	43.51	Item4	Identify an equivalent algebraic expression	6.EE.3	8.EE.7
2005	12	3	12	MC	Hard	Algebra	30.84	Item5	Determine the x-coordinate where 2 lines intersect	8.EE.8	8.EE.8
2005	12	4	11	MC	Medium	Algebra	48.35	Item6	Find x in the solution of a system of linear equations	8.EE.8	8.EE.8
2007	8	9	15	MC	Hard	Number properties and operations	33.9	Item7	Interpret number expressed in scientific notation (calculator available)	8.EE.4	8.EE.4
2007	8	11	11	MC	Hard	Algebra	25.04	Item8	Identify the graph of a linear equation	8.F.3	8.EE.6
2007	8	11	18	SCR	Easy	Algebra	74.13	Item9	Solve system of linear equations given in context	8.EE.8	8.EE.8
2009	8	10	1	MC	Easy	Algebra	68.37	Item10	Identify equation equivalent to given equation	6.EE.3	8.EE.7
2009	8	10	7	MC	Easy	Algebra	70.82	Item11	Identify solution from graph of linear equations	8.EE.8	8.EE.8
2009	12	2	14	SCR	Medium	Algebra	41.11	Item12	Solve a system of linear equations	8.EE.8	8.EE.8
2011	8	8	5	MC	Medium	Algebra	58.94	Item13	Identify a non-equivalent equation (calculator available)	6.EE.7	8.EE.7.
2011	8	12	7	MC	Hard	Algebra	31.38	Item14	Determine equation of a line given a point and the slope	XXXX	8.EE.6
2011	8	12	12	MC	Hard	Algebra	32.85	Item15	Find solution to equation	6.EE.5	8.EE.7.
#	#	#	#	#	#	#	#	Item16	SBAC Practice Item (20)	#	#
#	#	#	#	#	#	#	#	Item17	SBAC Practice Item (16)	#	#
#	#	#	#	#	#	#	#	Item18	SBAC Practice Item (16 and 24)	#	#

NAEP Content Area: Number properties and operations Question: Interpret number expressed in scientific notation (calculator available). Gr.8. 2007. Item7 Iowa CCSS classification: 8.EE.4; Alaska CCSS classification: 8.EE.4.	National Data:	MT Data:
<div><div>9.8413 02</div><p>The figure above represents a calculator display showing a number in scientific notation. That number is</p><p>A. 0.098413 B. 0.98413 C. 19.6826 D. 98.413 E. 984.13</p></div>	<div><p>Score</p><p>Correct 34%</p><p>Incorrect 64%</p><p>Omitted 2%</p><p>0 100</p><p>Percentage of Students</p></div>	<p>38% correct</p> <p>Answer: E</p>
NAEP Content Area: Number sense, properties, and operations Question: Identify decimal for scientific notation (calculator available). Gr.8. 2003. Item1 Iowa CCSS classification: 8.EE.4; Alaska CCSS classification: 8.EE.4.	National Data:	MT Data:
<p>The diameter of a red blood cell, in inches, is 3×10^{-4}. This expression is the same as which of the following numbers?</p> <p>A. 0.00003 B. 0.0003 C. 0.003 D. 3,000 E. 30,000</p>	<div><p>Score</p><p>Correct 41%</p><p>Incorrect 57%</p><p>Omitted 1%</p><p>0 100</p><p>Percentage of Students</p></div>	<p>48% correct</p> <p>Answer: B</p>
SBAC Practice Test Items, Item16		
Alaska CCSS classification: 8.EE.6		



Look at the graph of the linear equation.



Write an equation for the line in slope-intercept form.

← → ↶ ↷ ✖

1	2	3	x	y
4	5	6	+	- × ÷
7	8	9	< = >	
0	.	-	$\frac{\Box}{\Box}$	$\sqrt{\Box}$ $\sqrt[n]{\Box}$ π

For this item, a full-credit response (1 point) includes:

- a correct equation, such as $y = -\frac{1}{3}x$

NAEP Content Area: Algebra

Question: Determine equation of a line given a point and the slope. Gr.8. 2011. Item14

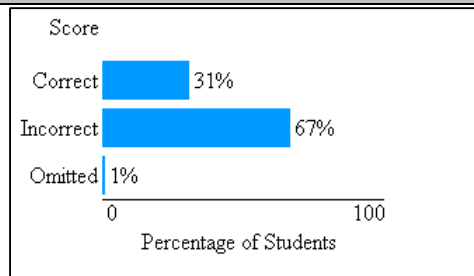
Iowa CCSS classification: XXXX; **Alaska CCSS classification:** 8.EE.6

Which of the following is an equation of a line that passes through the point (0, 5) and has a negative slope?

- A. $y = 5x$
- B. $y = 5x - 5$
- C. $y = 5x + 5$
- D. $y = -5x - 5$
- E. $y = -5x + 5$

National Data:

MT Data:



28% correct

Answer: E

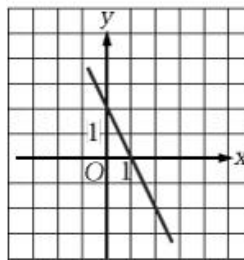
NAEP Content Area: Algebra

Question: Identify the graph of a linear equation. Gr.8. 2007. Item8

Iowa CCSS classification: 8.F.3; **Alaska CCSS classification:** 8.EE.6

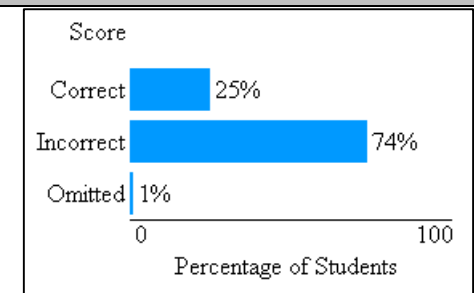
Which of the following is the graph of the line with the equation $y = -2x + 1$?

C.



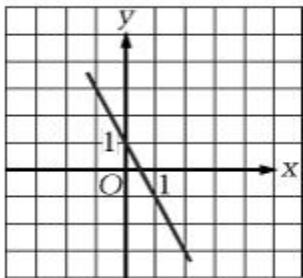
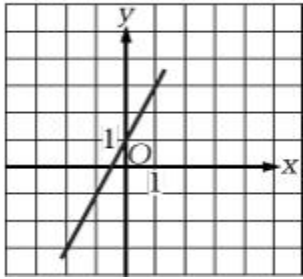
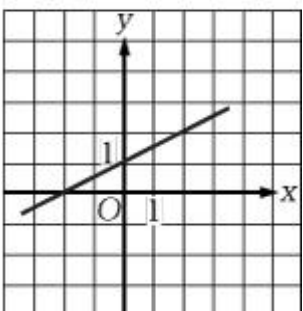
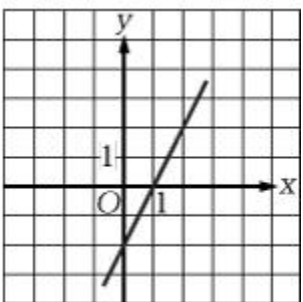
National Data:

MT Data:



20% correct

Answer: A

<p>A. </p> <p>B. </p>	<p>D. </p> <p>E. </p>		
--	--	--	--

SBAC Practice Test Items, Item17

Alaska CCSS classification: 8.EE.7

16

Drag numbers into the boxes to complete each equation with the given number of solutions.

0
1
2
3
4
5
6
7
8
9

Delete

A. Equation with no solutions

$$8x - 3x + 2 - x = \square x + \square$$

B. Equation with one solution

$$8x - 3x + 2 - x = \square x + \square$$

C. Equation with infinitely many solutions

$$8x - 3x + 2 - x = \square x + \square$$

For this item, a full-credit response (2 point) includes:

- an equation with a slope of 4 and an intercept that is not 2 for part A
AND
- an equation that does not have a slope of 4 for part B
AND
- an equation with a slope of 4 and an intercept of 2 for part C

For partial credit (1 point), the student correctly answers part B and either part A or part C.

For example,

- $8x - 3x + 2 - x = 4x + 3$
AND
- $8x - 3x + 2 - x = 3x + 3$
AND
- $8x - 3x + 2 - x = 4x + 2$

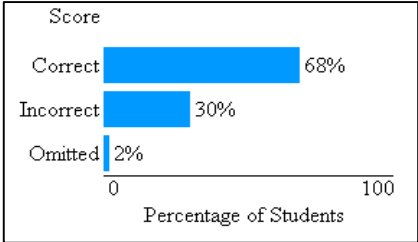
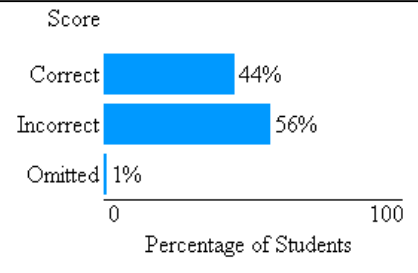
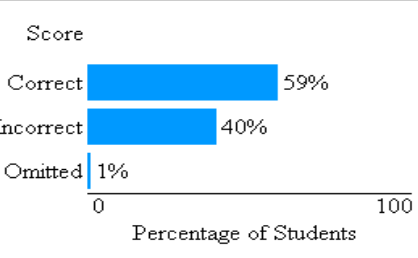
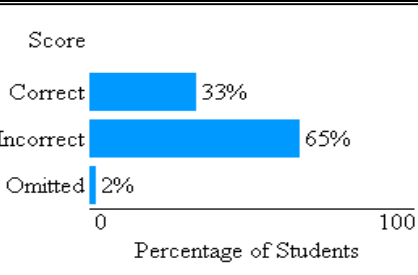
NAEP Content Area: Algebra

Question: Identify equation equivalent to given equation. Gr.8. 2009. Item10

Iowa CCSS classification: 6.EE.3; Alaska CCSS classification: 8.EE.7.

National Data:

MT Data:

Which of the following equations has the same solution as the equation $2x + 6 = 32$?	 <table><thead><tr><th>Score</th><th>Percentage of Students</th></tr></thead><tbody><tr><td>Correct</td><td>68%</td></tr><tr><td>Incorrect</td><td>30%</td></tr><tr><td>Omitted</td><td>2%</td></tr></tbody></table>	Score	Percentage of Students	Correct	68%	Incorrect	30%	Omitted	2%	68% correct Answer: E
Score	Percentage of Students									
Correct	68%									
Incorrect	30%									
Omitted	2%									
NAEP Content Area: Algebra Question: Identify an equivalent algebraic expression. Gr.8. 2005. Item4 Iowa CCSS classification: 6.EE.3; Alaska CCSS classification: 8.EE.7.	National Data:	MT Data:								
Which of the following is equal to $6(x + 6)$?	 <table><thead><tr><th>Score</th><th>Percentage of Students</th></tr></thead><tbody><tr><td>Correct</td><td>44%</td></tr><tr><td>Incorrect</td><td>56%</td></tr><tr><td>Omitted</td><td>1%</td></tr></tbody></table>	Score	Percentage of Students	Correct	44%	Incorrect	56%	Omitted	1%	44% correct Answer: D
Score	Percentage of Students									
Correct	44%									
Incorrect	56%									
Omitted	1%									
NAEP Content Area: Algebra Question: Identify a non-equivalent equation (calculator available). Gr.8. 2011. Item13 Iowa CCSS classification: 6.EE.7; Alaska CCSS classification: 8.EE.7.	National Data:	MT Data:								
Which of the following equations is NOT equivalent to the equation $n + 18 = 23$?	 <table><thead><tr><th>Score</th><th>Percentage of Students</th></tr></thead><tbody><tr><td>Correct</td><td>59%</td></tr><tr><td>Incorrect</td><td>40%</td></tr><tr><td>Omitted</td><td>1%</td></tr></tbody></table>	Score	Percentage of Students	Correct	59%	Incorrect	40%	Omitted	1%	65% correct Answer: A
Score	Percentage of Students									
Correct	59%									
Incorrect	40%									
Omitted	1%									
NAEP Content Area: Algebra Question: Find solution to equation. Gr.8. 2011. Item15 Iowa CCSS classification: 6.EE.5; Alaska CCSS classification: 8.EE.7.	National Data:	MT Data:								
The point $(4, k)$ is a solution to the equation $3x + 2y = 12$. What is the value of k ?	 <table><thead><tr><th>Score</th><th>Percentage of Students</th></tr></thead><tbody><tr><td>Correct</td><td>33%</td></tr><tr><td>Incorrect</td><td>65%</td></tr><tr><td>Omitted</td><td>2%</td></tr></tbody></table>	Score	Percentage of Students	Correct	33%	Incorrect	65%	Omitted	2%	30% correct Answer: B
Score	Percentage of Students									
Correct	33%									
Incorrect	65%									
Omitted	2%									

Alaska CCSS classification: 8.EE.8.

15



Joe solved this linear system correctly.

$$6x + 3y = 6$$

$$y = -2x + 2$$

These are the last two steps of his work.

$$6x - 6x + 6 = 6$$

$$6 = 6$$

Which statement about this linear system must be true?

- Ⓐ x must equal 6
- Ⓑ y must equal 6
- Ⓒ There is no solution to this system.
- Ⓓ There are infinitely many solutions to this system.

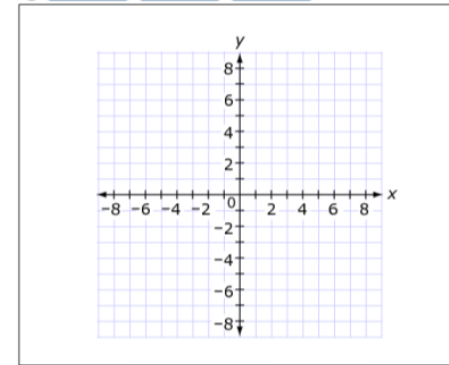
Answer: D

24



Use the Add Arrow tool to graph a system of two equations that has a single solution of $(-2, -3)$.

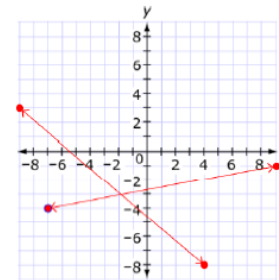
Delete Add Point Add Arrow



For this item, a full-credit response (1 point) includes:

- any 2 lines that intersect at the point $(-2, -3)$

For example,



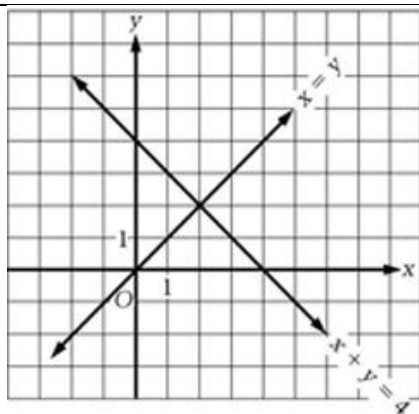
NAEP Content Area: Algebra

Question: Identify solution from graph of linear equations. Gr.8. 2009. Item11

Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.

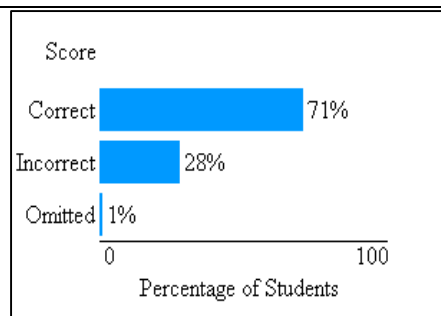
National Data:

MT Data:



Which point is the solution to both equations shown on the graph above?

- A. (0, 0)
- B. (0, 4)
- C. (1, 1)
- D. (2, 2)
- E. (4, 0)



74% correct

Answer: D

NAEP Content Area: Algebra and functions

Question: Solve problem involving two linear relationships.
Gr.8. 2003. Item3

Iowa CCSS classification: 8.EE.8; **Alaska CCSS classification:** 8.EE.8.

Key/Scoring Guide:

National Data:

MT Data:

Two large storage tanks, T and W, contain water. T starts losing water at the same time additional water starts flowing into W. The graph below shows the amount of water in each tank over a period of hours.

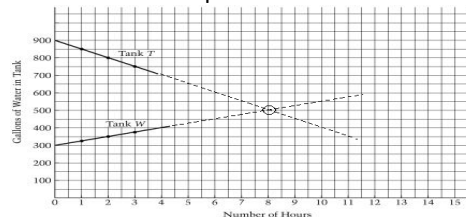
Scoring Guide

Sample Correct Responses:

8 hours.

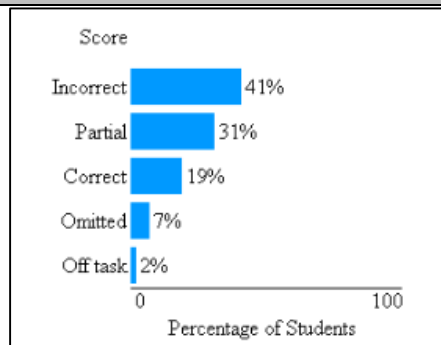
Graphical Solution:

Extend both lines to the right until they intersect. Then read the horizontal coordinate of the point of intersection.

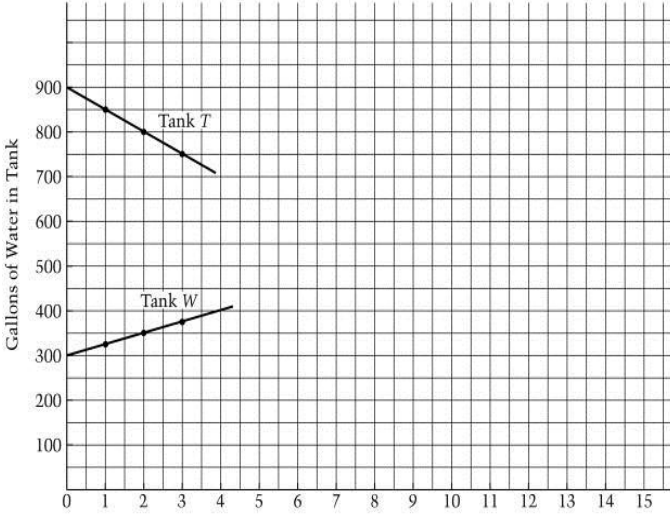
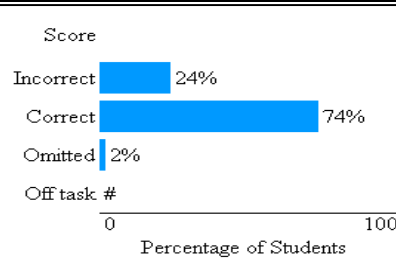


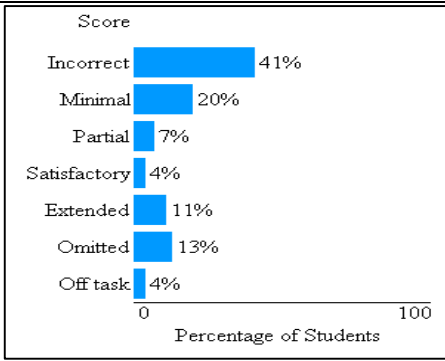
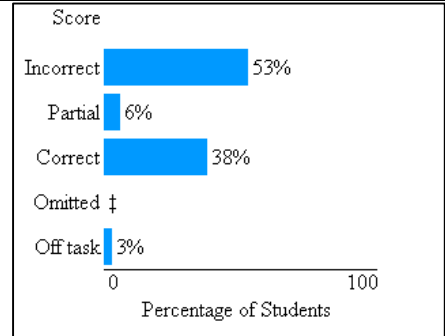
Note: If lines are extended correctly until they intersect, this will serve as sufficient justification. No words are required in this situation.

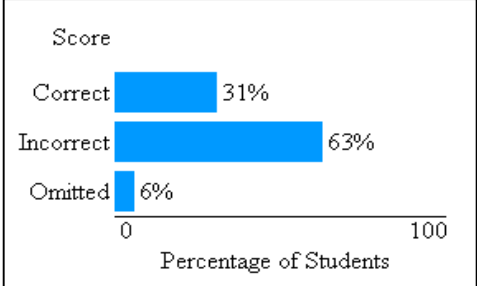
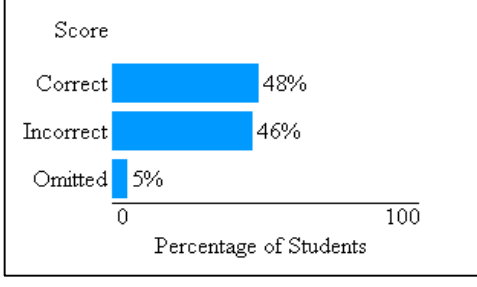
Algebraic Solution:



24% correct
38% partial

 <p>Assume that the rates of water loss and water gain continue as shown. At what number of hours will the amount of water in T be equal to the amount of water in W ?</p> <p>_____</p> <p>Show or explain how you found your answer.</p>	<p>Equation of lines: T $y = -50x + 900$ W $y = 25x + 300$ Point of intersection (8hrs, 500 gal)</p>		
<p>NAEP Content Area: Algebra</p> <p>Question: Solve system of linear equations given in context. Gr.8. 2007. Item9</p> <p>Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.</p> <p>At Jorge's local video store, "New Release" video rentals cost \$2.50 each and "Movie Classic" video rentals cost \$1.00 each (including tax). On Saturday evening, Jorge rented 5 videos and spent a total of \$8.00.</p> <p>How many of the 5 rentals were New Releases and how many were Movie Classics?</p> <p>New Releases_____Movie Classics_____</p>			<p>82% correct</p> <p>Answer: 2 new releases and 3 movie classics</p>
<p>NAEP Content Area: Algebra and functions</p> <p>Question: Solve problem using informal algebra (calculator available). Gr.8. 2003. Item2</p> <p>Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.</p>	<p>Key/Scoring Guide:</p>	<p>National Data:</p>	<p>MT Data:</p>

<p>While she was on vacation, Tara sent 14 friends either a letter or a postcard. She spent \$3.84 on postage. If it costs \$0.20 to mail a postcard and \$0.33 to mail a letter, how many letters did Tara send?</p> <p>Show what you did to get your answer.</p>	<p>Sample Correct Responses: 8 letters $.20(6) + .33(8) = \\$3.84$ Students may use a variety of strategies to solve this, including guess and check, formal algebra, or others. For example,</p> <table border="1"> <thead> <tr> <th># postcards</th><th># letters</th><th>total cost</th></tr> </thead> <tbody> <tr><td>1</td><td>13</td><td>4.49</td></tr> <tr><td>2</td><td>12</td><td>4.36</td></tr> <tr><td>3</td><td>11</td><td>4.23</td></tr> <tr><td>4</td><td>10</td><td>4.10</td></tr> <tr><td>5</td><td>9</td><td>3.97</td></tr> <tr><td>6</td><td>8</td><td>3.84</td></tr> <tr><td>7</td><td>7</td><td>3.71</td></tr> <tr><td>8</td><td>6</td><td>3.58</td></tr> </tbody> </table> <p>OR</p> $x + y = 14$ $.20x + .33y = 3.84$ <p>therefore,</p> $.20x + .33(14 - x) = 3.84$ <p>so $x = 6$ and $y = 8$</p>	# postcards	# letters	total cost	1	13	4.49	2	12	4.36	3	11	4.23	4	10	4.10	5	9	3.97	6	8	3.84	7	7	3.71	8	6	3.58	 <p>11% extended 7% satis. 9% partial</p>
# postcards	# letters	total cost																											
1	13	4.49																											
2	12	4.36																											
3	11	4.23																											
4	10	4.10																											
5	9	3.97																											
6	8	3.84																											
7	7	3.71																											
8	6	3.58																											
<p>NAEP Content Area: Algebra Question: Solve a system of linear equations. Gr.12. 2009. Item12 Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.</p> <p>What is the solution to the system of equations</p> $\begin{cases} 3x - 2y = -7 \\ x + y = 11 \end{cases} ?$ <p>Answer: $x = \underline{\hspace{2cm}}$ $y = \underline{\hspace{2cm}}$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: This is a grade 12 NAEP item but it is given as an example because it aligns to the new mathematics standard. Performance data is for 12th graders.</p> </div>	<p>National Data:</p>  <p>Scoring Guide Sample Correct Responses: Answer: $x = 3$ $y = 8$, Solution (not required in response):</p>	<p>MT Data:</p> <p>No state performance for 12th grade NAEP</p>																											
<p>NAEP Content Area: Algebra Question: Determine the x-coordinate where 2 lines intersect. Gr.12. 2005. Item5 Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.</p>	<p>National Data:</p>	<p>MT Data:</p>																											

$\begin{aligned}x + 2y &= 17 \\ x - 2y &= 3\end{aligned}$ <p>The graphs of the two equations shown above intersect at the point (x, y) What is the value of x at the point of intersection?</p> <p>A. $3\frac{1}{2}$ B. 5 C. 7 D. 10 E. 20</p> <div><p>Note: This is a grade 12 NAEP item but it is given as an example because it aligns to the new mathematics standard. Performance data is for 12th graders.</p></div>	 <table><caption>NAEP Score Data (Item 1)</caption><tr><th>Score</th><th>Percentage of Students</th></tr><tr><td>Correct</td><td>31%</td></tr><tr><td>Incorrect</td><td>63%</td></tr><tr><td>Omitted</td><td>6%</td></tr></table>	Score	Percentage of Students	Correct	31%	Incorrect	63%	Omitted	6%	No state performance for 12 th grade NAEP Answer: D
Score	Percentage of Students									
Correct	31%									
Incorrect	63%									
Omitted	6%									
NAEP Content Area: Algebra										
Question: Find x in the solution of a system of linear equations. Gr.12. 2005. Item6										
Iowa CCSS classification: 8.EE.8; Alaska CCSS classification: 8.EE.8.										
Gr. 12 $\begin{cases} x + 2y = 1 \\ 2x - y = 7 \end{cases}$ <p>In the solution of the system of equations above, what is the value of x?</p> <p>A. -1 B. 2 C. 3 D. 4 E. 5</p> <div><p>Note: This is a grade 12 NAEP item but it is given as an example because it aligns to the new mathematics standard. Performance data is for 12th graders.</p></div>	 <table><caption>NAEP Score Data (Item 2)</caption><tr><th>Score</th><th>Percentage of Students</th></tr><tr><td>Correct</td><td>48%</td></tr><tr><td>Incorrect</td><td>46%</td></tr><tr><td>Omitted</td><td>5%</td></tr></table>	Score	Percentage of Students	Correct	48%	Incorrect	46%	Omitted	5%	No state performance for 12 th grade NAEP Answer: C
Score	Percentage of Students									
Correct	48%									
Incorrect	46%									
Omitted	5%									